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APPLICATION OF ADVANCED FRACTURE MECHANICS TECHNOLOGY TO ENSURE--ETC(U)
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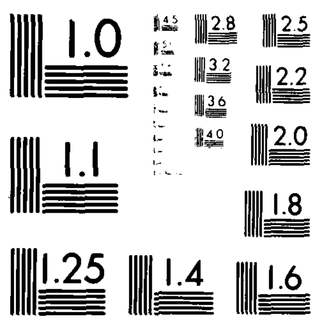

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APPLICATION OF ADVANCED FRACTURE MECHANICS
TECHNOLOGY TO ENSURE STRUCTURAL RELIABILITY
IN CRITICAL TITANIUM STRUCTURES

H. A. Ernst and J. D. Landes
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15 June 1981

Contract No. N00014-80-C-0655
Interim Report for Period 21 November 1980 - 31 March 1981

Prepared for
OFFICE OF NAVAL RESEARCH
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1. ABSTRACT

This report describes the progress in a program designed to assist the Navy in developing and applying advanced fracture mechanics technology to ensure structural reliability in critical applications of titanium alloys. Phase I of the program which was to gather information and data needed for applying advanced fracture mechanics technology to structural reliability analysis has been completed. The results of this phase are given in terms of a list of areas of concern which must be addressed in the future and a list of pertinent exerimental work already completed or in progress. Phase II of this program will use the above information to conduct structural integrity analyses of specific models. A methodology for structural integrity analysis is presented both for general service conditions and accident conditions, which gives an general outline for conducting structural integrity analysis. The method will be applied to specific models supplied by the Navy. A final section of this report describes a method of design for assessing stable crack propagation under load control conditions.

2. INTRODUCTION

(a) Background

The assurance of safe and reliable structural performance of critical components, structures, and equipment subjected to adverse loading conditions has always been a matter of vital concern to both the U.S. Navy and the Westinghouse Electric Corporation. The capability to conduct appropriate structural integrity analyses takes on an added importance when new equipment, designs, materials, inspections and fabrication procedures are concerned.

In these situations there is little or no service experience to rely upon; hence, a thorough structural integrity analysis, incorporating all of the interacting factors must be included as a major element in the overall plan. Such analyses should take advantage of the most advanced technology areas that are applicable to the situation of concern; in this case modern fracture mechanics technology offers a unique and directly applicable capability.

Early developments of fracture mechanics focused on plane-strain or essentially linear elastic fracture conditions (LEFM) and on relatively high strength brittle materials such as aircraft structures, missile cases, gun tubes, etc. Soon the technology was extended to include fatigue and stress corrosion crack propagation. Later on, because of the recognized limitations in the applicability of LEFM, effort was devoted to extend the fracture mechanics technology to encompass situations involving considerably more plasticity than is permissible under LEFM conditions. As a result the break through came in the form of the path independent J-integral, a field parameter analogous to K in LEFM. The general usefulness of the technology has thus been extended to a much broader spectrum of applications and

materials: lower strength, higher localized stress regions, low cycle fatigue and creep controlled crack growth. Even more recently, the technology has taken another major step forward with the advent of J resistance curves, tearing modulus concepts and tearing instability models. These recent developments offer the capability to predict the permisable amounts of stable crack growth in the ductile temperature regime, and the eventual instability conditions for the catastrophic failure of the structure by ductile tearing under fully plastic conditions. More importantly these recent advances in technology offer the promise of enabling the design of structures and selection of materials so as to avoid any possibility of failure due to ductile tearing instability.

In short, fracture mechanics provides engineers with a powerful new tool for more effective design pertaining to structural reliability; it therefore seems logical that it should be an important part of the Navy's titanium program.

(b) Scope of Work

I Objectives

The overall objective of this program is to assist the Navy in developing and applying advanced fracture mechanics technology to ensure structural integrity in critical applications of titanium alloys. To achieve this the following specific objectives are included:

1) Development of methods for assessing Structural Reliability

Different methods to assess structural reliability will be considered and the best possible choice will be proposed. This method should include the latest in fracture mechanics methodology.

2) Responding to Specific Navy Concerns

Should the Navy have at any given moment a specific concern, it could be included in the present program if budget time and general scope permits.

3) Recommendation of Methods for Implementing Structural Reliability Procedures

Once the overall methodology has been established, recommendations for implementing specific procedures will be made.

II. Approach

The program consists basically of two phases.

Phase I. Assimilation of pertinent information and data

In this phase a comprehensive review of the currently available information and data needed for applying advanced fracture mechanics technology to structural reliability analysis will be conducted. This review will include such pertinent information areas as loading conditions and stresses involved, material properties, fabrication procedures and nondestructive inspection.

Phase II. Structural Integrity Analysis

Phase II will include detailed structural integrity analyses of specific models using the best available input information and advanced fracture mechanics concepts. The specific models to be analyzed are to be subjected to mutual agreement between the Navy and Westinghouse technical personnel, consistent with the level of funding available. The most advanced concepts to date are to be applied to the model and loading conditions selected for analysis. In this selection of the model every attempt will be made to choose geometries for which appropriate K or J fracture mechanics expressions are already available. If appropriate expressions are not available to model the geometries of interest it will be necessary for either the Navy or Westinghouse to develop such expressions. The effort required to develop an elastic plastic expression for some complex geometry and loading conditions using a finite element approach is not included in the present plan. However if deemed necessary and appropriate such a task could be added to the Phase II work.

(c) Organization of Present Report

This report is organized as follows. In Section 3 the procedures used to achieve the mentioned objectives are explained in detail. In Section 4 the results of the work done are shown. First, the results of the survey of Phase I are discussed. Second a structural reliability methodology is proposed. Finally a stability analysis procedure is explained, and an example model is analyzed.

3. PROCEDURE

The approach used in order to accomplish the objectives mentioned above is as follows.

I) <u>Survey</u>

An extensive survey was conducted to gather information and data. This survey included visits to several different Navy facilities. The main points addressed in the survey were.

a) Navy concerns. An adequate characterization was made of the different areas that concern Navy personnel.

b) Available information and data. A list of areas where experimental data exist or is being generated today was made.

II) <u>Structural Reliability Methodology</u>

As a starting point of Phase II, some effort was devoted to organizing the structural reliability methodology to be followed. Several steps were identified.

- Development of method itself
- Identification of data required
- Determination of data and information areas which are missing
- Implementation of the methodology

4. RESULTS AND DISCUSSION

Survey: Navy concerns.

The first phase of this program is "Assimilation of pertinent information and data." To do this several meetings were held between members of the Westinghouse program team and Navy technical personnel at a number of different Navy facilities. Also Westinghouse participated in a ONR Ti-100 Workshop. As a result of these meetings, many areas were identified as having significance to the problem of structural integrity analysis for Ti-100 and questions were raised regarding these areas; a list of those follows.

1) Toughness - How much toughness is enough? How should toughness be characterized for structural analysis?
2) Dynamic Loading - What is the fracture behavior under dynamic loading rates? Can the parameters used for conventional fracture analysis also be used under dynamic loading?
3) Fatigue - Can fatigue to failure be analyzed? What is the effect of zero to compression loading on fatigue crack growth analysis?
4) Low Cycle Fatigue - What is the effect of this on crack initiation and growth?
5) Failure Criteria - No failure criterion is presently used. Can one be identified particularly for analyzing fatigue to failure?
6) Crack Growth Under Sustained Load - This phenomenon has been observed in the form of subcritical cracking and delayed failure. Is this related to environmental influences, creep or time dependent fracture toughness behavior?

7) Effect of Prestrain on Fracture Behavior - How is fracture toughness and other fracture behavior influenced by an initial prestrain?

8) Effect of Residual Stresses in Welds - How can these be measured and how can they be incorporated into a structural integrity analysis?

9) Thickness Effects - How are these accounted for in structural analysis?

10) Scale Models vs. Real Structures - How well do scale models predict fracture behavior in real structures? What models are the most appropriate ones to analyze?

11) Shop Fabrication vs. Field Fabrication - Are properties in field welded structures as good as those in shop welded structures?

12) Explosion Bulge and Tear Tests - What significance do these have; Can they be analyzed using advanced fracture approaches?

<u>Survey: Data available or work in progress</u>

In these meetings the present availability of experimental data and the work in progress in different areas were discussed. A list of those follows:

1) Dynamic vs Static J-R Curves

The effect of loading rate in the J-R curve.

2) J-R Curve and Toughness

Material toughness data and the possible effect of variables on those values, i.e., thickness, prestrain,etc.

3) Fatigue

Data fatigue crack growth for different conditions.

4) Stress corrosion cracking

Differenc aspects regarding the stress corrosion cracking

5) Sustained Load Crack Growth

Crack growth under a fixed load level in different conditions.

6) Creep

Studies and experimental work in creep and creep crack growth.

7) Environmental Effects

The effect of environment on all of the above mentioned areas.

Result of the Survey

The specific results of the survey are listed above, however some general conclusions can be drawn

1) Qualitative vs Quantitative

The concerns and questions raised on the several issues are not quantitative rather they are of a qualitative nature. The question after is whether some parameter would at all affect the result, rather than precisely how much. This shows that many concerns are of a basic nature.

2) Diversity of Concerns

The survey revealed that at different Navy locations and among people of different technical functions there was frequently a difference in interests, concerns and priorities.

3) Points Address Experimental Work

Most of the points of concern can only be answered by additional experimental work.

4) Points of Concern and Basic Research

Several points that were raised are subjects presently being addressed in basic fracture research. In fact, it is not just a question of how to apply known concepts to titanium, or how titanium behaves under certain conditions compared to other materials extensively tested. Instead, several points of concern are still an open question in other areas where fracture mechanics is much more advanced and has been extensively tested.

Structural Reliability Methodology

Method

As was mentioned before as a starting point of Phase II, effort was devoted to organize the methodology needed to assess structural reliability. A flow chart was constructed for accident and service conditions, showing the different steps and questions to decide upon. The chart is shown in Figure 1.

Accident conditions are characterized by severe loading of short duration. As a result the main concern is the toughness (J-Resistance) of the material under those conditions, as opposed to fatigue, sustained load cracking, etc.

The first step suggested is to take the J-R curve from the bending specimen. Reason: There is experimental evidence that these are conservative when compared with tension type specimens. The alternative possibility is to test other geometries and in fact more work has to be done in the area. The second step is to take the J-R curve from static loading. Reason: It seems that it is conservative compared to dynamic loading. The alternative possibility is to try to test at the actual rates of interest. Finally in the third step it is suggested to take a critical J, J_C (from the J-R curve) that can be either the J_{IC} or J at maximum load of the specimen tested. Reason: The tearing modulus T ($T = \frac{E}{\sigma o^2} dJ/da$) is often too low to utilize the benefit of the extra toughness due to crack growth, and the J at maximum load in a laboratory specimen is expected to be conservative as compared with the J at the corresponding maximum load in a structure. The alternative possibility is to design to assure stable crack growth - (See next sections).

In conclusion, this chart shows a simple way of deciding upon different issues, central column, the reason why in the left column and a way of improving the analysis, right column.

Service conditions are characterized by cycled loading, long time exposure of the crack to the environment, and long periods of sustained load. The corresponding flow chart is shown in the bottom of Figure 1.

The first step is to identify the loading spectrum, load amplitude frequency, etc. With this information and the material data available the number of cycles to failure can be calculated. The critical crack length is obtained from the material toughness K_{Ic} or J_C. Eventually the effect of reloading on toughness should be incorporated to better assess the critical crack length as well as the effect of time on material properties. Finally, the effect of environment on each of the previously mentioned mechanisms has to be determined to give as a final result the largest initial crack length admissible in the structure compatible with inspection capabilities.

Data Needed

To implement the correct structural reliability methodology, a number of subjects have to be addressed and experimental data obtained. Some of them are currently under investigation, some are well documented, and some need additional work. A list of all of them follows.

- Bending vs Tension -- Effects of geometry on the J-R curve.
- Loading Rates -- Effects of loading rates on the J-R curve.
- Reloading -- Effects of strain history on the J-R curve
- Part Through Cracks -- Material response and failure analysis with part through cracks
- Environment -- Effect of environment in all of the above plus sustained load cracking.
- Creep
- Crack arrest
- Inspection capabilities

o Effect of other variables on the J-R curve -- Residual Stresses, Prestrain, Thickness-Ligament Proportion, Heat affected zones, Welds, Material variability

As was mentioned this is a list of items that should be addressed to correctly implement the structural reliability methodology. Part of them have been or are being addressed at present.

The flow chart of Figure 1 offers the possibility of attacking the structural reliability problem for accident conditions at a relatively simple level (central column) or at a more sophisticated one right column. In this spirit and in the next section, the procedure for the stability analysis of a structure is fully explained and the fracture proof design concept is introduced and applied to an example model.

Stability Analysis

Even though the potential of the J-integral [1-5] as the governing parameter of the crack tip stress-strain field was established earlier, the question of stable vs unstable crack growth in the ductile tearing mode remained unresolved until very recently. In fact, a model capable of predicting stable/unstable behavior taking into account specimen geometry, a/W ratio, material properties and overall behavior of the structure was simple non-existent.

Unstable crack growth can be regarded as a lack of balance between an externally applied crack drive force and material crack growth resistance. Instability will ensure when the rate of increase of the applied drive force exceeds that of the material resistance to crack growth.

Recently, the basic implications of this concept were further explored by Paris et al. [6,7] and as a result, it was demonstrated that the overall characteristics of the structure plays a major role in instability and its effects have to be included in the rate balance

mentioned. In this work, they introduced a non-dimensional quantity called the tearing modulus, T, that in general has the form:

$$T = \frac{E}{\sigma_o^2} \frac{dJ}{da} \tag{1}$$

where E is the elastic modulus and σ_o is the flow stress. If Equation (1) is evaluated using the J-Resistance curve of the material, the resulting T is the material tearing modulus T_{mat}. If instead, dJ/da in Equation (1) is calculated as the rate of change of crack drive or the applied J, per unit virtual crack extension, with the condition of total displacement, δ_{tot}, kept constant (or other similar conditions specified), the resulting T is the applied tearing modulus T_{app}. And so following References [6-7], instability will occur when:

$$T_{app} > T_{mat} \tag{2}$$

Using the condition of total displacement constant, the compliance of the structure, C_M, is introduced into the analysis and T_{app} becomes a function of C_M. Consequently, according to the theory, Equation (2) instability is predicted provided the values of T_{mat} and expressions for T_{app} are known. In their original work, Paris et al also performed the first experimental evaluation of the theory. In tests of three point bend specimens loaded in series with a spring bar of adjustable length, the compliance of the system, C_M, was varied from test to test, producing stable or unstable behavior in complete agreement with the theory. In this work the expressions of T_{app} for different configurations were calculated by assuming that the material was perfectly plastic and that the crack was growing under limit load conditions.

Later, Hutchinson and Paris [8] presented a more general expression for T_{app} for a specimen loaded in series with a spring, Figure 2, simulating the structure

$$T_{app} = \frac{E}{\sigma_o^2} \left| \left.\frac{\partial J}{\partial a}\right|_P - \left.\frac{\partial J}{\partial P}\right|_a \left.\frac{\partial \delta}{\partial a}\right|_P \frac{1}{\left[C_M + \frac{\partial \delta}{\partial P}\right]_a} \right| \tag{3}$$

where δ is the displacement due to the crack and C_M is the compliance of the spring. All the terms appearing in the above equation are calibration functions, i.e., they don't bear any information regarding the material response to crack growth. These functions can be obtained from finite element analysis or experimentally from blunt notch specimen tests, and no "real" crack growth test is needed for their determination. This scheme has been used [9-10] to obtain T_{app} for different configurations of practical interest and instability predicted by comparing the value of T_{app} obtained from Equation (3) with the experimentally obtained T_{mat}.

The stability problem can be also approached from a different point of view, emphasizing the role of the load vs displacement (P-δ) test record. In fact, as shown by Ernst et al [11-12], both T_{mat} and T_{app} can be evaluated from a single specimen test record and the conditions for instability can be found directly.

The T_{mat} is defined as the rate of change of J with respect to crack length along the J-R curve, or actual test record.

$$T_{mat} = \frac{E}{\sigma_o^2} \left(\frac{dJ}{da}\right)_{mat} = \frac{E}{\sigma_o^2} \left(\frac{\partial J}{\partial a} + \frac{\partial J}{\partial \delta} \frac{\partial \delta}{\partial a}\right)_{mat} \tag{4}$$

At the same time, considering the load P as a function of displacement, δ, and crack length, a,

$$P = P(\delta, a) \tag{5}$$

it can be differentiated to give

$$dP = \left.\frac{\partial P}{\partial a}\right|_{\delta} da + \left.\frac{\partial P}{\partial \delta}\right|_{a} d\delta \tag{6}$$

or rearranging

$$\frac{d\delta}{da} = \frac{\left.\frac{\partial P}{\partial a}\right|_{\delta}}{\frac{dP}{d\delta} - \left.\frac{\partial P}{\partial \delta}\right|_{a}} \tag{7}$$

where the terms $\partial P/\partial a|\delta$ and $\partial P/\partial\delta|_a$ are calibration functions, and the term $dP/d\delta$ is to be measured from the actual test record. Replacing Equation (7) in Equation (4) and noting that:

$$T_{mat} = \frac{E}{\sigma_o^2}\left|\left.\frac{\partial J}{\partial a}\right|_{\delta} + \left.\frac{\partial J}{\partial \delta}\right|_{\partial}^{2} \frac{1}{\left.\frac{\partial P}{\partial \delta}\right|_{a} - \frac{dP}{d\delta}}\right| \tag{8}$$

which is a general expression for T_{mat}.

The applied tearing modulus, T_{app}, is defined as the rate of change of J_{app} with crack length under the condition that the overall displacement is kept constant (or equivalent condition). Thus, T_{app} is given by

$$T_{app} = \frac{E}{\sigma_o^2}\left.\frac{dJ}{da}\right|_{\delta_{tot}} = \frac{E}{\sigma_o^2}\left|\frac{\partial J}{\partial a} + \frac{\partial J}{\partial \delta}\left.\frac{d\delta}{da}\right|_{\delta_{tot}}\right| \tag{9}$$

The condition δ_{tot} = constant, is equivalent to $d\delta_{tot} = 0$, or separating the total displacement into a part due to the rest of the structure (uncracked body part), δ_M

$$d\delta_{tot} = d\delta + d\delta_M = 0 \qquad d\delta_{tot} = d\delta + d\delta_M = 0 \tag{10}$$

$$= d\delta + C_M dP = 0 \tag{11}$$

where $C_M = (K_M)^{-1} = d\delta_M/dP$ can be associated with the linear compliance of the system (spring + testing machine + uncracked specimen). T_{app} can be then calculated using Equations (6), (9), (10) and (11) to give

$$T_{app} = \frac{E}{\sigma_o^2} \left| \left.\frac{\partial J}{\partial a}\right|_\delta + \left.\frac{\partial J}{\partial \delta}\right|_a^2 \frac{1}{\left.\frac{\partial P}{\partial \delta}\right|_a + K_M} \right| \tag{12}$$

An alternative expression can also be found for T_{app} by combining Eqs. (8) and (12) giving

$$T_{app} = \frac{E}{\sigma_o^2} \left| \left.\frac{\partial J}{\partial a}\right|_\delta + \left.\frac{\partial J}{\partial \delta}\right|_a^2 \frac{1}{K_M + \frac{dP}{d\delta} + \frac{(\partial J/\partial \delta)^{-2}}{\frac{\sigma_o^2}{E} T_{mat} - \left.\frac{\partial J}{\partial a}\right|}} \right| \tag{13}$$

Moreover, the calibration functions $\partial J/\partial a|_\delta$ and $\partial J/\partial \delta|\alpha$ can be expressed in terms of current values of J and P respectively if the expression for P in Equation (5) is known. As an example, as discussed in [12] for the compact specimen:

$$\left.\frac{\partial J}{\partial a}\right|_\delta = -\frac{\gamma J}{b} \tag{14}$$

$$\left.\frac{\partial J}{\partial \delta}\right|_a = \eta \frac{P}{b} \tag{15}$$

where

$$\begin{aligned} \gamma &= 1 + .76\ b/w \\ \eta &= 2 + .522\ b/w \end{aligned} \tag{16}$$

As a result, Equations (8) and (12) with the corresponding condition of Equations (14) and (15), allow us to obtain T_{mat} and T_{app} as a function of current values of J, P and b, the slope $dP/d\delta$ and the stiffness of the structure K_M; all quantities obtainable from a single

test record [12]. Furthermore, the expressions for T_{mat} and T_{app} can be compared giving the condition for unstability

$$T_{app} > T_{mat} \tag{17}$$

if and only if

$$-dP/d\delta > K_M = C_M^{-1}$$

The condition for instability can be obtained using a different approach.

Consider the P-δ record of a bend specimen tested under displacement control and the corresponding calibration (nongrowing crack curves as shown in Figure 3.

Suppose now that an identical specimen (same a/W) is tested this time in series with a spring, Figure 2. It can be seen that the effect of the spring on the calibration curves and is the test record is just to shift every point in Figure 3 to the right by an amount

$$\delta_M = PC_M \tag{18}$$

with the new displacements $\delta A'$ and $\delta B'$ given by

$$\begin{aligned} \delta_{A'} &= \delta_A + P_A C_M \\ \delta_{B'} &= \delta_B + P_B C_M \end{aligned} \tag{19}$$

where P_A and P_B are the loads at points [A-A'] and [B-B'], respectively. Note that corresponding points [A-A'], [B-B'], etc. have the same value of J (J depending on a/W and displacement only due to the crack, δ). Thus the resulting test record is expected to go through these corres-

ponding points [A'], [B'], etc. in order to follow the J-R curve as before. Combining Equations (19) gives

$$\delta_{B'} - \delta_{A'} = \delta_B - \delta_A + C_M(P_B - P_A) \tag{20}$$

Note that for the portion of the test record where $P_B - P_A < 0$ (dropping part), the relative distance of subsequent points is diminished by the addition of the spring.

$$\delta_B - \delta_A > \delta_{B'} - \delta_{A'} \quad \text{if } P_B < P_A \tag{21}$$

In fact, if enough compliance is added, this relative distance could even turn to be negative; i.e., point [B'] lying to the left of [A']. If this is the case the test record would have to go backwards (in δ) to pass through [B'] in order to follow the J-R curve. But this is not compatible with the boundary condition which asks for a monotonically increasing displacement. Thus, the test record gets as near to point [B'] as it is allowed to (vertical drop), corresponding to unstable growth.

The conditions for stability can be then expressed as

$$\begin{aligned} \delta_{B'} - \delta_{A'} &< 0 \quad \text{unstable} \\ \delta_{B'} - \delta_{A'} &> 0 \quad \text{stable} \end{aligned} \tag{22}$$

Replacing Equations (19) and (20) in the above expression gives

$$\frac{\partial \delta}{\partial P} < - C_M \tag{23}$$

$$\text{or} \quad -\frac{dP}{d\delta} > K_M = C_M^{-1} \quad \text{for instability.}$$

It can be seen from Equation (23) that under the specified testing conditions, instability cannot occur in the portion of the P-δ record where the load is increasing ($dP/d\delta > 0$ would imply $C_M < 0$). The value of C_M which satisfies Equation (23) can be interpreted as the necessary compliance to be added to the system in order to set subsequent points to the same displacement causing them instability.

As a result, by just inspecting the test record of a certain specimen geometry (with [illegible]hout the spring) the amount of additional structural complianc[illegible] to cause instability in an identical specimen test, or [illegible] additional compliance C_{RC} (remaining compliance capacity) [illegible]an be readily obtained.

$$C_{RC} = tg\ \alpha = -dP/d\delta \tag{24}$$

where α is shown in Figure 3.

So far, it is obvious that instability can be predicted by the C_{RC} concept for a given specimen geometry. Nevertheless, it is of much greater interest to be able to predict instability for a certain specimen without actually running a test. This also can be accomplished using the C_{RC} concept [12, 14]. In fact, $- dP/d\delta$ can be obtained from Equation (8) and substituted in Equation (24) to get

$$[C_{RC}]^{-1} = -\frac{dP}{d\delta} = -\left.\frac{\partial P}{\partial \delta}\right|_a + \left\{\frac{\partial P}{\partial a}\right\}^2 \frac{1}{\dfrac{\sigma_o^2}{E} T_{mat} - \left.\dfrac{\partial J}{\partial a}\right|_\delta} \tag{25}$$

as the instability condition for any configuration. Note that as mentioned, $\partial P/\partial\delta|_a$, $\partial P/\partial a|_\delta$ and $\partial J/\partial a|_\delta$ are calibration functions and the only additional information needed is T_{mat} which comes from other specimen tests. Thus, instability can be predicted in a given configuration with no more than calibration functions and universal material

properties (no actual test needed). This approach is analogous to the one that was used by several investigators [9-10]. In fact, if the material resistance curve is available (obtained from a certain specimen geometry) the P-δ record for an untested configuration can be predicted using the procedure of Shih et al [11-13] and the instability condition obtained following Equations (23) and (25). Alternatively, with the material resistance curve information and the calibration functions known, the crack drive force, $J = J(\delta_{tot}, a)$, can be obtained. The tangency condition can be found, and instability predicted.

It is emphasized here that all of the above makes use of the concept that a universal geometry-independent crack growth resistance material property can be found. In situations where the universal parameter does not exist as such, some investigators have adopted the value of T_{mat} obtained by testing bend type specimens as a conservative lower bound estimation. Nevertheless, the apparent variability of the J-R curve due to different geometries is still an open question which needs further exploration.

<u>Stable crack growth under load control conditions. Example Model.</u>

In the last section the crack instability analysis methodology in terms of the tearing modulus T was explained in detail, specialized to fixed displacement systems where the system compliance (spring in series with the specimen) was shown to exert an influence on instability occurrences. This specialization is not confining, however, in the sense that the T approach can be applied to either load or displacement controlled systems. [15, 16]

To better illustrate the method consider the example model shown in Figure 4. It consists of a thin wall cylinder with circumferential stiffners a distance W apart. The defect is a longitudinal through crack located at the center of the panel. The model is deformed under load control conditions by means of internal pressure which is shared by both the cracked body and stiffness. This system can be modelled by the

specimen in parallel with a spring (not in series as before) as shown in Figure 5. In this case the total applied load P_{tot} is balanced by the load at the specimen, P, plus the load at the spring, P_M,

$$P_{tot} = P + P_M \tag{26}$$

Whereas the displacement is the same for both members

$$\delta_{tot} = \delta = \delta_M \tag{27}$$

The condition for stability can be found following an approach similar to that one of the previous section.

Consider the P-δ record of a specimen (tested under displacement control) and the corresponding non-growing crack calibration functions as shown in Figure 6. It is well known that if the same specimen is tested under load control conditions, the crack will grow in an unstable fashion at the point of maximum load P_{max} (or where the slope of the P-δ record starts to be negative). But if instead, an identical specimen is tested in parallel with a spring of stiffness K_M, under load control, the effect of the spring on the calibration curves and test record is just to shift every point, in Figure 6a, up in load by an amount

$$P_M = \delta K_M \tag{28}$$

Giving for the total load of two generic points A and B located at displacements δ_A and δ_B respectively

$$\begin{aligned} P_{totA} &= P_A + \delta_A K_M \\ P_{totB} &= P_B + \delta_B K_M \end{aligned} \tag{29}$$

Assuming $\delta_B > \delta_A$ stability will be guaranteed if

$$P_{totB} > P_{totA} \tag{30}$$

or using Equation (29), the stability condition can be found to be

$$P_{totB} - P_{totA} = (P_B - P_A) + K_M(\delta_B - \delta_A) > 0$$

$$K_M > - \frac{P_B - P_A}{\delta_B - \delta_A} = - \frac{dP}{d\delta} \tag{31}$$

Note that this condition is exactly the same as that one of Equation (23) for completely different conditions; as a result, for a spring in series under displacement control as well as a spring in parallel under load control, stability will be guaranteed if and only if

$$K_M > - dP/d\delta$$

This allows full use of the methodology presented in the previous section for this example model. In fact Eq. (25) can be used again here to obtain the condition on K_M to guarantee stable behavior

$$K_M > - \frac{dP}{d\delta} = \left| - \left.\frac{\partial P}{\partial \delta}\right|_a + \left.\frac{\partial P}{\partial \delta}\right|_a^2 \frac{1}{\frac{\sigma_o^2}{E} T_{mat} - \left.\frac{\partial J}{\partial a}\right|_\delta} \right| \tag{25}$$

Note that as mentioned before $\partial P/\partial\delta|_a$, $\partial P/\partial a|_\delta$ and $\partial J/\partial a|_\delta$ are just calibration functions and the only additional information needed is T_{mat} which can be obtained from other specimen tests. As a result instability can be predicted in a given geometry without actually having to test it, but just using calibration functions and universal material resistance properties.

5. CONCLUSIONS

A survey was conducted among Navy personnel to gather information and data available regarding the structural reliability analysis. The survey revealed that the concerns and questions raised were not quantitative but rather of the qualitative nature and that among people of different technical functions there was frequently a difference in interests, concerns and priorities. The survey showed that many of the points of concern need experimental work and also that some of the issues are still a subject of basic research in areas where fracture mechanics is much more advanced and has been extensively tested.

As a starting point of Phase II - Structural Reliability Analysis -- a flow chart was constructed for accident and service conditions. The chart shows a simple way of deciding upon different issues, a justification for doing so and a way of improving the analysis.

Finally, a way of implementing the stability methodology is explained in detail and a new design philosophy for assessing stable crack propagation under load control conditions is developed.

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7. STATEMENT OF WORK

The proposed work is a 16 month effort directed at assisting the Navy in developing and applying advanced fracture mechanics to ensure structural integrity in critical applications of titanium alloys. The program consists of two major phases.

Phase I - Assimilation of Pertinent Information and Data

A compcehensive review will be concluded to assimilate the currently available information and data needed for applying advanced fracture mechanics technology to structural reliability analyses of critical components or structures using titanium alloys.

Phase II- Structural Integrity Analyses

Detailed structural integrity analyses of specific models of components or structures will be conducted using the best available input information and data and the most advanced state-of-the-art fracture mechanics concepts.

The original proposed schedule called for a 24 month period of performance, this schedule is given in Table A. The program was then reduced to 16 months. Recently a request has been made to extend the period of performance 6 months from a September 30, 1980 to a March 30, 1982 termination date. The tasks remaining in Phase II along with an approximate schedule is given in Table B.

8. PROGRAM SCHEDULE

Table A. Original Program Schedule

	Task	Months after start of contract: 6 — 12 — 18 — 24
PHASE I:	Review, evaluate, and assimilate pertinent data and information	Δ———Δ
I-1.	Initial discussion to identify areas and available information	—Δ
I-2.	Identification of tentative models	Δ—Δ
I-3.	Detailed review of information and data	Δ——Δ
I-4.	Evaluation of information and preparation of report	Δ—Δ
PHASE II:	Structural Integrity Analyses	Δ—————Δ
II-1.	Identification of specific models	Δ—Δ
II-2.	Initial analyses	Δ—Δ
II-3.	Refined analyses	Δ—Δ
II-4.	Final report	Δ—Δ

Table B - Remaining Phase II Tasks with Approximate Schedule

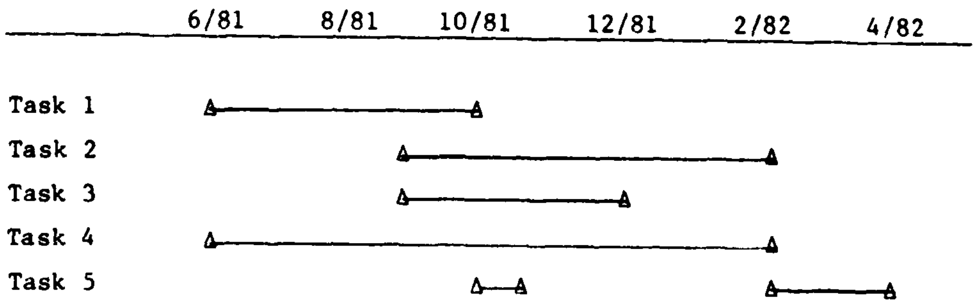
1. Analysis of the structural model provided by DTNSRDC, Carderock, with various crack locations and for conventional loading.

2. Incorporation of dynamic loading considerations into the analysis of this tructural model.

3. Incorporation of mode II fracture considerations into the above analysis.

4. Incorporation of new approaches to elastic-plastic fracture Characterization into the above analysis.

5. Reporting:
The approximate schedule is given in the following table:



LIST OF FIGURES

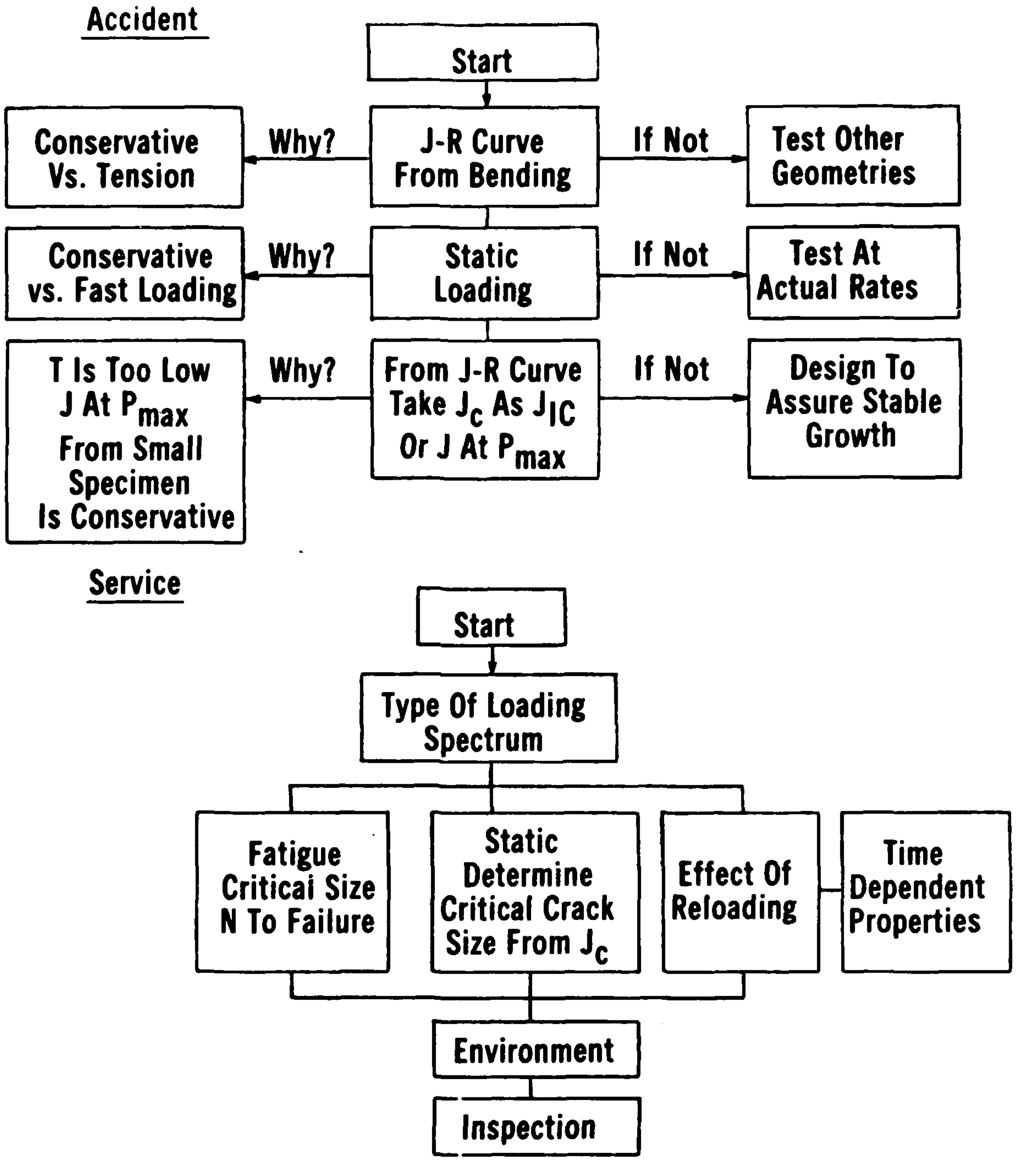


Fig. 1 - Structural Reliability Methodology Flow Chart

Dwg. 7746A98

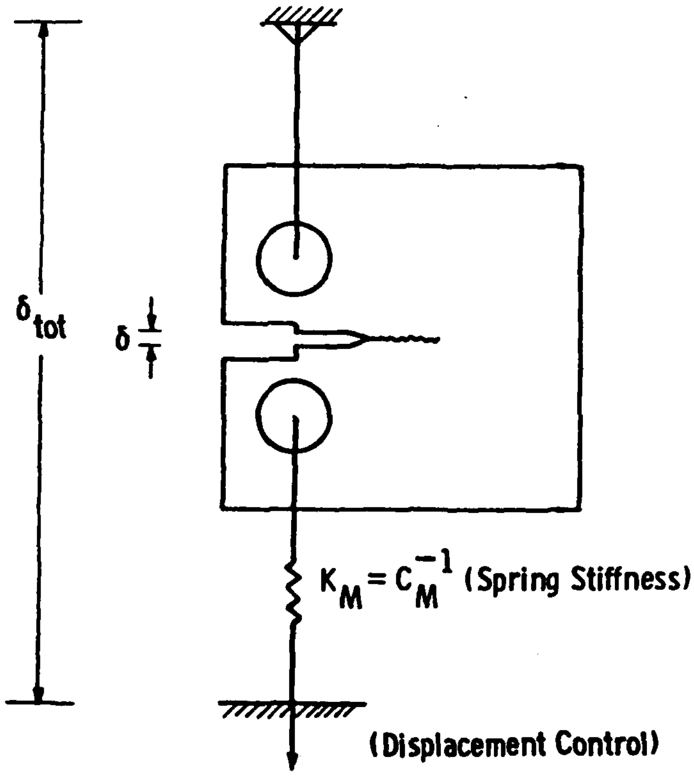
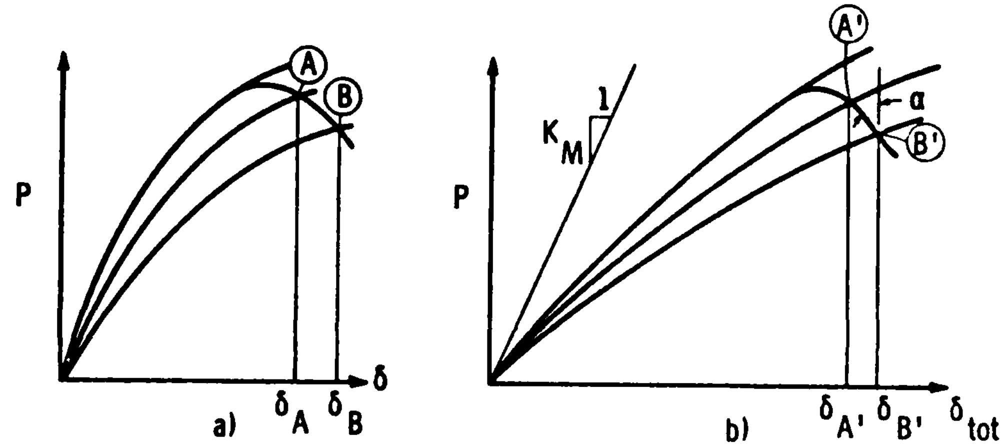


Fig. 2 — Displacement control test of a bend specimen in series with a spring bar

Curve 727169-A

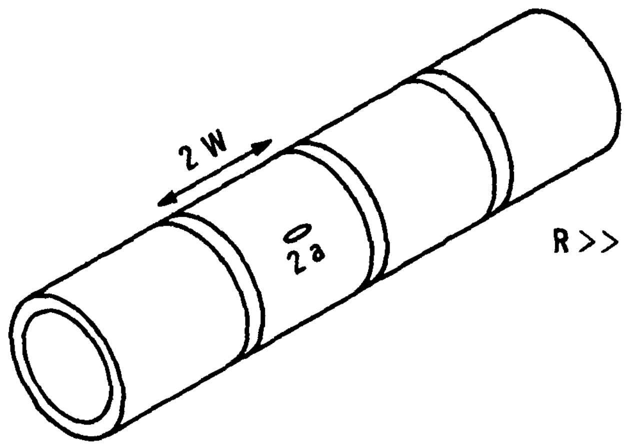


Vertical Drop $\left(-\frac{dP}{d\delta_{tot}} \rightarrow \infty\right)$ is Obtained

when $K_M = -\frac{dP}{d\delta}$

Fig. 3 — P-δ record of a specimen a) without and b) with spring in series.

Dwg. 7740A43



Geometry : Thin Walled Cylinder (R >> t) with Circumferential Stiffners 2 W apart

Defect: Longitudinal - Through - Crack Symmetrically Located on the Panel

Loading: Internal Pressure P

Fig. 4 — Cylinder example model.

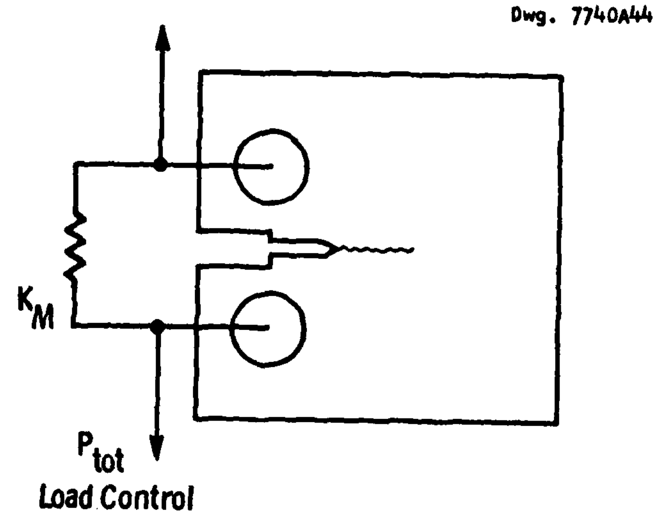


Fig. 5 — Specimen in parallel with a spring in a load-control test

Curve 728788-A

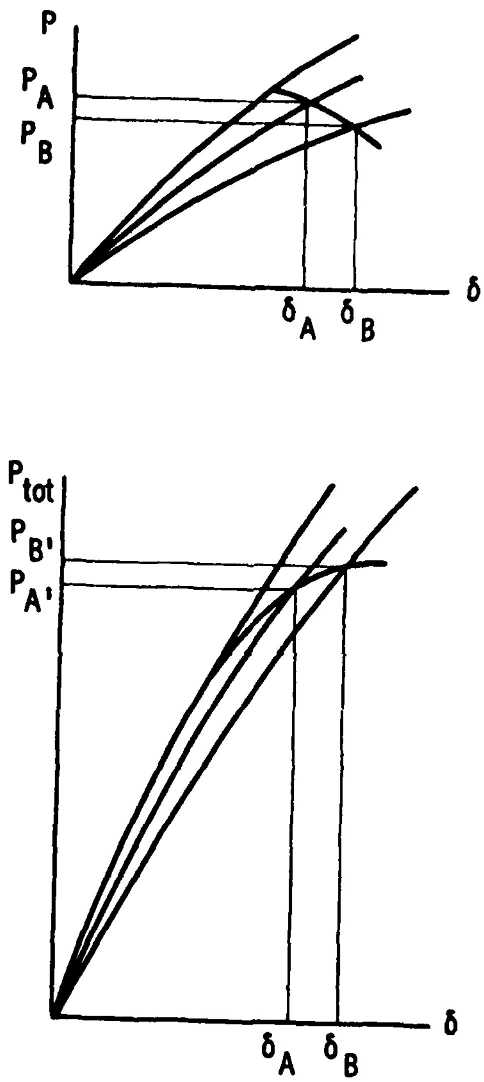


Fig. 6 — P-δ records of a specimen a) with and b) without a spring in parallel

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